**FAST TRACK PAPER**

**Linear or non-linear rheology in the Earth’s mantle: the prevalence of power-law creep in the postglacial isostatic readjustment of Laurentia**

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**SUMMARY**

The great majority of postglacial rebound computations carried out during the last three decades assumed a purely linear rheological relation for the mantle. Experimental data on high-temperature creep deformation and modelling of other tectonic processes, however, might also support the existence of non-linear creep mechanisms. We addressed postglacial rebound in North America through an axially symmetric finite-element model with a composite (linear plus non-linear) mantle rheology. In such a formulation, the transition stress $\sigma_T$ governs the balance between linear and non-linear creep components, while the term $\sigma_B$, added to the effective shear stress, accounts for the background (ambient) stress induced by convection and other tectonic processes. By varying $\sigma_T$ and $\sigma_B$ in the ranges 0–10 MPa and 0–5 MPa respectively, we found that composite models fit Relative Sea Level (RSL) variations at 29 North American sites better than the purely linear model. On the basis of the effective shear stress induced in the mantle by glacial forcing (1–3 MPa), our results indicate that power-law creep accounts for the majority of the strain rate.

**Key words:** Laurentide ice sheet, non-linear rheology, postglacial rebound, power-law creep, relative sea-level variations.

**INTRODUCTION**

The redistribution of ice and water mass on the Earth’s surface, together with the corresponding readjustment of mantle material inside the Earth during the last 20 kyr, is the source of anomalies on a number of geophysical observables that can be used to infer the rheological properties of the Earth’s interior. Most previous investigations (Haskell 1935; Cathles 1975; Peltier & Andrews 1976; Yuen et al. 1986; Tushingham & Peltier 1991, 1992) were carried out under the simple hypothesis that the relation between stress and strain rate is linear. That means assuming diffusion of vacancies across grains (Nabarro–Herring creep) or along grain boundaries (Coble creep) (Ranalli 1995) to be the dominant creep mechanisms in mantle polycrystalline aggregates. In this case, the mantle would behave mainly as a highly viscous Newtonian fluid, and its governing rheological relation (accounting also for elastic properties of mantle rocks) would be the well-known Maxwell body equation

$$\dot{\varepsilon}_{ij} = \frac{1}{2\eta} \sigma_{ij} + \frac{1}{2\mu} \dot{\sigma}_{ij},$$

(1)

where $\dot{\varepsilon}_{ij}$ and $\sigma_{ij}$ are the deviatoric strain-rate and stress tensors respectively, $\eta$ is the Newtonian viscosity, and $\mu$ is the shear modulus. However, both microphysical studies and laboratory experiments suggest that at least one other mechanism, the climb of dislocations inside the grains, might explain the long-term deformation of mantle materials. Owing to the relation between climb velocity and stress, this mechanism follows a non-linear stress–strain equation of the type (Ranalli 1995)

$$\dot{\varepsilon}_{ij} = A\sigma_{ij}^{n-1}\sigma_{ij},$$

(2)

where $\sigma_{ij}$ is the effective shear stress, $n$ the power-law exponent, and $A$ is a material parameter that depends on temperature and pressure. Since diffusion creep is strongly dependent on grain size, while dislocation creep depends on the effective stress, the assumption of linearity is only valid if the grain size is small and the stress low: the exact values of these two parameters in the mantle are not precisely known. Theoretical and experimental evidence, however, suggest that the two creep mechanisms almost balance (Ranalli 1998). Their interplay can be properly described by the concept of transition stress ($\sigma_T$); that is, the amount of stress at which the total strain-rate contributions from diffusion and dislocation creep are the same. If the stress exceeds the transition stress then the creep in the mantle mainly follows a power law; otherwise, it behaves linearly.

Since no general analytical methods for simulating the deformation of a power-law medium subject to a surface load have been derived yet, the investigators who analytically considered
postglacial rebound with a power-law mantle rheology made a number of simplifying assumptions (Post & Griggs 1973; Brennen 1974; Crough 1977). It is only during the last 10 years that some works have employed the finite-element (FE) numerical technique to model the relaxation of a purely non-linear medium (Wu 1992, 1993, 1995, 1999, 2001, 2002), or of a composite (linear plus non-linear) one (Gasperini et al. 1992; Giunchi & Spada 2000) under the effect of the glacial load.

Excluding the first attempt (Wu 1992), cast in an oversimplified plane-strain geometry that substantially ruled out the possibility that any part of the mantle could be non-Newtonian, all the successive works using a purely non-linear medium (Wu 1993, 1995, 1999, 2001, 2002) allowed the existence of non-linear portions inside the mantle, even though they confirmed the incompatibility of RSL data with a fully non-linear mantle in the absence of ambient stress. Recently, Wu (2001) found that a purely non-linear mantle with \( A \approx 3 \times 10^{-35} \text{ Pa}^{-3} \text{ s}^{-1} \) and an ambient stress level of around 10 MPa is able to explain the sea-level data both outside and inside the Laurentian ice margin.

A different approach (Gasperini et al. 1992; Giunchi & Spada 2000) considered a composite rheology in which the value of the transition stress \( \sigma_T \) regulates the simultaneous contributions of linear and non-linear creep mechanisms to the total strain rate. Gasperini et al. (1992) also included an additive term \( \sigma_B \) to the effective shear stress \( \sigma_E \) of eq. (2), accounting for the background (ambient) stress induced by other geodynamic processes active in both the lithosphere (plate tectonics) and the mantle (thermal convection). On the basis of convection-associated stress fields in a non-Newtonian upper mantle overlying a Newtonian lower mantle (van den Berg et al. 1991), this approximation seems to acceptably simulate the ambient stress effect. The expression for the resulting flow law is

\[
\dot{\epsilon}_{ij} = \frac{1}{2\eta^*} \left[ 1 + \left( \frac{\sigma_E + \sigma_B}{\sigma_T} \right)^{n-1} \right] \sigma_{ij} + \frac{1}{2\mu} \sigma_{ij}, \tag{3}
\]

where \( \eta^* \) is the viscosity of the Newtonian component, while the non-linear creep coefficient is given by \( A = 1/2\eta^* \sigma_T^{n-1} \).

The application of this composite flow law for the upper mantle showed (Gasperini et al. 1992) that it is possible to reproduce well

Figure 1. Comparison of different FE codes. Computed vertical displacement as a function of time at three sites located at the centre of the load \((R = 0)\), close to the largest horizontal ice extension \((R = 2200)\) at the Last Glacial Maximum (LGM), and farther \((R = 2550)\). Dotted lines refer to the Marc 2001 commercial code, while small triangles refer to the modified version of the Tecton code used in this work.

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the vertical displacement and velocity time evolution of a purely Newtonian mantle for both the Fennoscandia and Laurentide ice sheets, for a wide range of reasonable parameter values. These results (Gasperini et al. 1992) led to the conclusion that postglacial rebound is not able to discern steady-state creep in the upper mantle.

Unfortunately, the paper by Gasperini et al. (1992) was not followed by a deeper analysis, and thus some points remained unaddressed: the computations were performed by a self-modified version (Gasperini & Sabadini 1989; Gasperini et al. 1992) of the simple FE code Tecton (Melosh & Raefsky 1980), which might not work properly; the assumed deglaciation history was oversimplified; no comparisons with real uplift data were done; the fit between linear and non-linear models was only evaluated qualitatively; the comparisons among models mainly concerned the centre of the load, and only a few cases were shown in the ‘sea-level transition zone’, close to the border of the ice sheet at the Last Glacial Maximum (LGM).


Assuming the rheological formulation of eq. (3), we explored the parameter space by quantitatively comparing the results of FE simulations with a data set of RSL data in North America. For most computations we employed the above-mentioned code Tecton (Melosh & Raefsky 1980), modified to include buoyancy (Gasperini & Sabadini 1989) and composite rheology (Gasperini et al. 1992), but we tested its reliability by rerunning some of the models on the commercial code Marc 2001 (MSC 2001). We chose to use Tecton because it is considerably faster than Marc (on a 1-GHz PC, a single run, with about 120 time steps on a grid of 40 by 82 rectangular linear elements, takes about 30 s with Tecton as opposed to 160–180 s with Marc), as well as being more easily included in minimization schemes as it is available as source code. The results of the comparison between Tecton and Marc (Fig. 1) indicate a very nice fit between the two codes, making us confident that past (Gasperini et al. 1992) and present computations done using our version of Tecton are correct. For the sake of computational efficiency, we again adopted an axisymmetric flat geometry. It has been demonstrated (Amelung & Wolf 1994; Wu & Johnston 1998) that, even for an ice load of the size of Laurentide, relative vertical displacement differences between flat and spherical models lie within observational uncertainties for sites located within the ice sheet or not very far from it. In any case, we expect the effects of our geometrical simplifications to be small with regard to the comparisons of linear versus non-linear models.

As glacial forcing we adopted the ICE-3G deglaciation history (Tushingham & Peltier 1991, 1992), which is the most recently published glacial model. Successive models such as ICE-4G and ICE-5G are only available on the Internet or through circulation among investigators and have not yet been published in a peer-reviewed journal. In order to adapt ICE-3G to our axisymmetric geometry, we first averaged the coordinates of the ICE-3G disc loads—weighted with the corresponding ice mass—to compute Laurentide ice-sheet barycentre coordinates at each time step. Then we determined the maximum radius and the maximum height of circular parabolic glaciers having the same volume as the sum of all the North American disc loads at each time step. The barycentre of the whole process, located at 58.9N, 87.7W, is considered the centre of the axisymmetric model for comparisons with RSL data (black circle in Fig. 2). The RSL sequences are estimated from our numerical model by simply adding the eustatic variation term to the computed relative displacement. We evaluated the goodness-of-fit of models with respect to RSL data using the following misfit function (Wu 1999):

\[
X^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{rsl^i_{\text{err}} - rsl^i_{\text{vis}}}{} \right)^2,
\]

where \(rsl^i_{\text{err}}\) and \(rsl^i_{\text{vis}}\) are, respectively, the observed and computed relative sea levels and the corresponding errors of each single RSL observation, and \(N\) is the total number of observations.

Following Wu (1999), we used a set of 29 North American sites (black stars in Fig. 2), which can be considered the most reliable among those included in the RSL global database by Tushingham & Peltier (1992). In addition, we limited our comparison to the last 8 kyr BP, when other terms of the complete sea-level equation (Wu & Peltier 1983) can be neglected. Our composite models include an elastic lithosphere of 120 km and a uniform viscoelastic mantle of...
2770 km with power-law creep exponent \( n = 3 \). Elastic parameters and density vary with depth according to the Preliminary Reference Earth Model (Dziewonski & Anderson 1981). We also built some models having the same geometrical and mechanical characteristics, but with a purely linear mantle flow law.

For composite rheology models we varied both \( \sigma_T \) and \( \sigma_B \) on a grid (of about 1000 nodes) in the ranges 0–10 MPa for \( \sigma_T \) and 0–5 MPa for \( \sigma_B \). At each grid node we computed the value of the viscosity \( \eta^* \) of the Newtonian component minimizing the misfit function of eq. (4). For linear models the only free parameter was the value of the true Newtonian viscosity \( \eta \). To estimate the best-fitting \( \eta^* \) and \( \eta \) values we built a Fortran minimization code including Tecton as a subroutine. As a minimization algorithm we used the IMSL routine UVMGS (IMSL 1991), which implements the golden section search technique. Fig. 3 shows the misfit behaviour as a function of \( \eta \) and \( \eta^* \) for (a) the purely Newtonian model and (b) a composite rheology model with \( \sigma_T = 1.5 \) MPa and \( \sigma_B = 1.6 \) MPa. The whole minimization procedure required a run of more than 14 000 models and took about two weeks on a 1-Ghz PC. In Fig. 4 the light grey surface and the contours indicate the lowest misfit values for a composite model with varying \( \sigma_T \) and \( \sigma_B \), while the dark grey plane indicates the level of the lowest misfit (\( \chi^2 = 13.1 \)) obtained by a purely linear model.

**DISCUSSION AND CONCLUSIONS**

For every combination of \( \sigma_T \) and \( \sigma_B \), the composite rheology fits RSL data better than the linear one. The relative difference between the misfit values of composite and linear models ranges from about 5 per cent, close to the origin of the horizontal axes (\( \chi^2 = 12.4 \) at \( \sigma_T = 0 \) and \( \sigma_B = 0 \)), to 30–40 per cent, around the absolute minimum of the light grey surface (\( \chi^2 = 8.3 \), reached at \( \sigma_T = 1.5 \) MPa and \( \sigma_B = 1.6 \) MPa). For \( \sigma_T \gg 1.5 \) MPa the misfit function tends asymptotically to the linear value. At the absolute minimum, the viscosity of the Newtonian component is \( \eta^* = 8.3 \times 10^{21} \) Pa s, while the non-linear creep coefficient results in \( A = 2.7 \times 10^{-25} \) Pa s\(^{-1} \).

Owing to the smoothness of the misfit surface close to the minimum, we have to consider the corresponding best-fitting values of \( \sigma_T \) and \( \sigma_B \) just as very rough estimates of the true values of these parameters in the mantle. However, on substituting them into eq. (3) and considering that in our models the effective shear stress induced in the mantle by glacial forcing ranges from 1 to 3 MPa, the power-law creep component accounts for 75 to 95 per cent of the total strain rate. The predominance of the non-linear component might even be higher if the transition stress is lower than 1 MPa, as suggested by experiments on polycrystalline olivine (Ranalli 1998). This

**Figure 4.** Comparison of best-fit misfit functions for composite and linear models. The light grey surface and the contours represent the entire set of best-fitting composite models, while the dark grey plane indicates the level of the best misfit function (\( \chi^2 = 13.1 \)) obtained by a purely linear model. Two stars indicate the exact locations of the absolute minimum for composite models (\( \chi^2 = 8.3 \), at \( \sigma_T = 1.5 \) MPa and \( \sigma_B = 1.6 \) MPa).
evidence suggests that power-law creep is likely to be the dominant flow mechanism in the mantle.

On the basis of the previously cited literature (dominated by purely linear modelling) this finding might appear surprising, not least because the glacial forcing we used was developed under the assumption of a linear mantle rheology. In particular, our results are at odds with previous similar comparisons cast with a purely non-linear formulation and in the absence of ambient stress (Wu 1993, 1995, 1999, 2001, 2002). We must note, however, that the apparent contradiction could be explained by the different formulation of the models, as well as by an insufficient exploration of the parameter space in previous analyses. Fig. 5, concerning a purely non-linear model with zero background stress, shows how the misfit function goes below the best linear value (indicated by a horizontal line) for parameter \( A \) only in the range between \( 5 \times 10^{-10} \) and \( 5 \times 10^{-14} \) Pa\(^{-3}\) s\(^{-1}\), a range which includes the best-fitting value obtained by Wu (2001). Considering the experimental uncertainties and the simplifications made, the small difference in the misfit function does not allow us to assert that the purely non-linear model is definitely better than the purely linear one. We can instead confidently give a clear preference to the composite model with transition and/or background stress in the range 1–3 MPa.

We could infer that the linear component of our composite rheology, although accounting for a minority of the flow, represents a stabilizing factor, mitigating power-law rheology singularities. Even the relatively good fit found for several models with alternating linear and non-linear layers (Wu 1993, 1995, 1999, 2001, 2002) can be interpreted in terms of the same stabilizing action exerted by linear layers on non-linear ones. Moreover, the still rather unknown role of ambient stress, although not strictly necessary to the good fit of the composite models, might significantly contribute to ‘linearizing’ the response of the mantle to glacial load, thus making the non-linear term less sensitive to the variation of the glacially induced stress perturbations.

It has been suggested that the assumption of a non-linear rheology implies a stratification of the effective viscosity in the mantle that is usually reproduced – in linear models – by an increase of viscosity with depth. In the literature, the factor between the lower and upper mantle viscosities ranges from 2–4 (Wu & Peltier 1983; Tushingham & Peltier 1991, 1992) to 10–100 (Nakada & Lambeck 1987). Although a fair comparison should require the introduction of a similar layered structure even in the composite model, we tested a further linear model in which the viscosity is forced to increase below the mantle transition zone (at a depth of 670 km) by a factor of 15 (approximately the geometric average of the two extremes). The best-fitting stratified linear model (\( \eta_{UM} = 2.5 \times 10^{21} \) Pa s and \( \eta_{LM} = 3.8 \times 10^{22} \) Pa s) still gives a higher misfit (\( \chi^2 = 10.6 \)) than the best composite one (\( \chi^2 = 8.3 \)). The analysis-of-variance test (Draper & Smith 1981) gives \( F = 49.9 \) (with two numerator and 179 denominator degrees of freedom). This indicates that the variance reduction (22 per cent) of the best composite model (having three free parameters) with respect to the two-layer linear one (having only one free parameter) is significant with a confidence level >99 per cent. Even though we are well aware of the limits of our simplified approach, we believe that the consequences of our computations could be quite relevant as they overturn the current outlook of postglacial rebound modelling. In particular, the value of linear viscosity, estimated in previous works, could now appear to be the fortuitous result of the interplay of several physical quantities rather than a true material parameter of mantle rocks.

This work suggests a number of future investigations that could be performed to confirm the appropriateness of this rheological formulation: first of all, the inclusion in the modelling of other ice sheets and a more realistic geometry (spherical) and ice models (2-D), as well as application to other data sets (gravitational potential, horizontal deformations etc.); secondly, the addition of a vertical stratification of the rheological properties, based on current microphysical models (Ranalli 1995, 1998), which might improve the description of the material properties of the Earth’s mantle.

Our findings also reopen a number of geophysical questions that could be re-evaluated in the light of this approach. These include the modelling of the time and space evolution of the glacial load (actually deduced under the purely linear hypothesis), which might lead to a significant re-evaluation of the current uplift rates in deglaciated areas, with significant consequences for the estimated rate of global sea-level rise.

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