Mantle Rheology and Satellite Signatures From Present-Day Glacial Forcings

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We study the changes of the long-wavelength portion of the Earth's gravity field as a result of present-day glacial discharges and the possible growth of the Antarctic ice sheet. We employ both Maxwell and Burgers' body rheologies and find that there are significant differences in the responses between the two rheologies for time spans less than a century. Both present-day glacial forcing and the ice buildup on Antarctica cause non-negligible perturbations in \( J_2 \) of about one-third the observed amount. The contributions from the two mechanisms have opposite signs in \( J_2 \). Only for \( l \leq 3 \) and \( 6 \) do these cryospheric contributors act in concert with one another in disturbing the gravity field. All other degree harmonics, up to \( l = 9 \), have opposite signs from these two excitations. For a steady-state lower-mantle viscosity of \( 10^{22} \) Pa s, \( J_2 \) can attain a maximum value of around \( 1.5 \times 10^{-11} \) yr \(^{-1} \) and \( J_2 \) a value of \(-1.3 \times 10^{-11} \) yr \(^{-1} \) for current forcings. These have signs opposite to those predicted for Pleistocene deglaciation. All harmonics, up to \( l = 9 \), are sensitive to variations of the steady-state viscosity in the lower mantle, even for time scales shorter than 100 years. It is proposed that detailed satellite monitoring of present-day ice movements in conjunction with geodetic satellite missions will eventually provide a viable alternative for estimating deep mantle viscosity, useful in mantle convection models.

1. INTRODUCTION

Until the last few years estimates of mantle viscosity have been based upon the traditional method of analysis of postglacial rebound signatures, ranging from uplift to rotational data [O'Connell, 1971; Cathles, 1975; Peltier and Andrews, 1976; Lambeck and Nakiboglu, 1983; Sabadini and Peltier, 1981; Yuen et al., 1982, 1986; Wu and Peltier, 1982, 1983, 1984; Peltier, 1984; Yuen and Sabadini, 1985; Sabadini et al., 1985a]. Most of these analyses have been conducted in the spatial domain in which the observations are compared with theoretical predictions derived from viscoelastic models. Recently this analysis has been carried out also in the spectral domain for geoid anomalies. Recent studies of long-wavelength geoid anomalies and the assumed forcing by internal density heterogeneities inferred from seismic tomographic analyses [Hager et al., 1985; Hager and Clayton, 1987] have shown that this is a distinctly viable method for estimating the viscosity contrasts in the mantle. However, there are still contributors to the gravity field from pressure gradients in the upper mantle produced by large-scale plate motion [Hager and O'Connell, 1979; Ricard et al., 1986], whose influences on the inference of viscosity still need to be properly assessed. These three different ways of inferring mantle viscosity rely upon internal or external types of forcings with timescales longer than \( 10^4 \) years. Information about the deglaciation excitations function can be obtained from geological field observations [e.g., Andrews and Barry, 1978], while the locations of the density anomalies used to calculate the geoid perturbations are derived from seismic tomography [e.g., Dziewonski, 1984].

From observations of the orbital node of the LAGEOS satellite, temporal variations of the degree two coefficient of the gravity field, \( \delta J_2(t) \), have been found [Yoder et al., 1983]. The coefficient of the first term in this series is called \( J_2 = d(\delta J_2)/dt \). A value of \( J_2 = (0.35 \pm 0.03) \times 10^{-10} \) yr \(^{-1} \) is derived by Yoder et al. [1983]. Another source of surface forcing potentially useful for investigating the rheological structure is that due to recent glacial discharges [Meier, 1984], as was first pointed out by Yoder and Ivins [1985]. Recently, Wagner and McAdoo [1986] discuss a variety of time variations in the gravity field that may be observed in the future with satellites. It has been demonstrated by Gasperini et al. [1986] that recent glacial movements could produce a variation of the gravitational harmonic coefficient \( J_2 \) with an opposite sign and about one-third of the observed amount from analyses of LAGEOS orbital data [Yoder et al., 1983; Rubincam, 1984]. More interesting is the finding [Gasperini et al., 1986] that if, indeed the Antarctic ice sheet is currently growing [DOE, 1985], then part of the observed \( J_2 \) signal can be explained by invoking such an increase of ice mass in Antarctica. Hence the changes in ice-covered areas can deform the Earth's surface and induce significant and observable changes in the Earth's gravity field, much greater than has previously been surmised. Since this particular type of surface forcing is deterministic in nature, one may use it for learning about short-term rheology, much more so than large earthquakes [Sabadini et al., 1985b] whose temporal and spatial distributions are stochastic and, therefore, do not allow reliable means of coverage by satellite observations.

Observations of the Earth's surface from continuing flights of space shuttles and satellites can provide us with new
measurements of the mass balance of polar ice sheets and active glaciers; this stream of data can be converted into input for the surface forcing function to be used in calculating the transient responses of the planet. Precise tracking of satellite motions [Tapley et al., 1985] can be used to determine the gravitational forces acting on satellites. Future, more accurate geodetic satellite missions may also be able to determine the secular variations in the low degree and order geopotential coefficients. We can potentially learn much more about the mantle’s rheological behavior by taking advantage of satellite geodesy, satellite altimetry, and satellite imagery [e.g., Elachi et al., 1986] in light of this recent finding of the efficacy of slight disequilibria of ice masses in disturbing the Earth’s gravitational potential.

The purpose of this paper is to point out the importance of monitoring a range of long-wavelength harmonics of the gravity field by space techniques and to show that, even for a timespan of $10^5$ yr, it is still feasible to place bounds on the long-term viscosity of the lower mantle. An advantage of this approach is that, with advances in space technology, the surface forcing function from glacial melting can be known to great precision. In section 2 we describe the quantitative model used for predicting the secular variations of the gravitational potential and the new features of incorporating glacial forcings. The results for the excitation of the degree two harmonic are presented in section 3. The effects of the higher zonal harmonics are then discussed in section 4. Our conclusions along with some remarks on the potential of this new approach make up the final section.

2. Model Description and Theoretical Calculations of $J_2$

Calculations reported here are carried out following the analytical development given by Yuen et al. [1982, 1986] and Sabadini et al. [1982, 1984]. In this viscoelastic, self-gravitating, spherical Earth model we assume that the planet is not subject either to net external torque from atmospheric circulations [Barnes et al., 1983] or to magnetic torques at the core-mantle boundary [Madden and Le Mouël, 1982; Gire et al., 1984] during the recent changes of climate [Jones et al., 1986] which result in some mass transfer between glaciers and the oceans. The total angular momentum of the Earth is thus assumed to be constant. In this paper we will be primarily concerned with the external temporal changes of the gravity field due to viscoelastic relaxation of the Earth’s mantle in adjusting to slowly varying surface loads, such as glaciers, whose dynamical timescales are short in comparison to the ice-age cycle, $10^4$ yr. A closed hydrological cycle between the glaciers and oceans is valid for timescales exceeding a few decades. To calculate these transient responses of the displacement and gravity fields in a self-consistent manner, we solve a sixth-order ordinary differential system for each $l$ by an essentially analytical method, based on propagator methods. A linear rheology is used because of its mathematical simplicity and its proven ability to fit a wide range of geophysical data. Our theoretical formulation can handle all types of linear viscoelastic rheologies, ranging from the Maxwell [Sabadini et al., 1982] to the Burger’s body rheology [Yuen et al., 1986].

We will focus on the use of the Burger’s body rheology because of its versatility in describing the dynamics of both intermediate and long-term creep phenomena. This linear rheology is simple in that it is completely specified by the short- and long-term viscosities, $\nu_2$ and $\nu_1$, and by the relaxed and unrelaxed shear moduli, $\mu_2$ and $\mu_1$ [Yuen and Peltier, 1982]. Through the use of the correspondence principle the shear modulus of the Burgers’ body rheology in the Laplace transformed domain contains all of the physical attributes, such as transient and steady-state creep. This simplifies greatly the solution of the mixed initial- and boundary-value problem associated with viscoelastic processes. For the Burger’s body rheology the transformed rigidity $\mu(s)$ is given [Yuen and Peltier, 1982] as

$$\mu(s) = \frac{\mu_2(s + \mu_2/\nu_2)}{s^2 + [(\mu_2 + \mu_3)/\nu_2 + \mu_2/\nu_1]s + (\mu_2/\nu_1^2)}$$

(1)

In (1) one finds explicitly how these four parameters enter into the transformed constitutive relation and how the Burger’s body model would help to interpret the postglacial rebound data. In the long time limit ($s \to 0$) the shear modulus $\mu(s)$ becomes $\nu_1$ as in the Maxwell case so that the long-term viscosity $\nu_1$ governs the relaxation times for greater than $\nu_1/\mu_2$ and $\nu_2/\mu_2$. Other types of transient and steady state rheologies based on generalized retardation spectrum have been proposed by Mueller [1986].

From (1) it is easy to see that the Burger’s body rheology can be reduced to an equivalent Maxwell rheological model for three regimes (1) $\nu_2 \ll \mu_2$, (2) $\nu_2 \gg \nu_1$, (3) $\nu_1 \ll \nu_2$. However, recent laboratory work of transient rheology on the extraction of these parameters in terms of Burger’s body [Smith and Carpenter, 1987] shows that none of these three asymptotic limits is valid for crustal and upper-mantle minerals. For the range of parameter values $\nu_2/\mu_2$ and $\nu_2/\nu_1$ inferred from laboratory studies, simplifications to equivalent Maxwell models are not possible and recourse must inevitably be made to solving the appropriate boundary-value problem for Burger’s body rheology [e.g., Ranalli, 1987].

It is surprising that rheological laws used for the lower mantle seldom refer to its composition, which is generally accepted to consist principally of MgSiO_3 of perovskite structure [Liu, 1979]. The perovskite phase is estimated to constitute about 70% of the lower mantle mass. Work on analogue systems of MgSiO_3 perovskite [Poirier et al., 1983] shows the possibility of Newtonian creep in the lower mantle and a low activation volume. Recent laboratory experiments (Knittle and Jeanloz, 1987; Heinz and Jeanloz, 1987) of (Mg, Fe)SiO_3 also support the idea of very little pressure-dependence in the rheology of potential lower mantle substances. However, there is still no published work done on extracting transient creep data for both perovskite and periclase, which are done under adiabatic and high-pressure conditions.

In order to study these short timescale phenomena, we will use the Burger’s body rheology because of its ability to capture transient dynamics much more faithfully than the Maxwell model. We have constructed a new four-layer model consisting of an elastic lithosphere, a two-layer Burger’s body mantle divided up at the 670 km discontinuity and an inviscid core. The physical parameters are given in Table 1. This model differs from that used in Gasperini et al. [1986] and Yuen et al. [1986] in that the upper mantle, as well as the lower mantle, now has Burgers’ body rheology. This additional complication increases the number of normal modes to 13 for each angular order $l$ in models where internal buoyancy, due to chemical layering or to incomplete phase transitions over short timescales [Christensen, 1985], is present at the 670 km discontinuity.

The properties of the radial short-term viscoelastic structure can be found in the eigenspectra which consist of the inverse
relaxation times: \([s_i]\) and the residues associated with the different types of boundary conditions, such as surface or tidal loadings. In Table 2 are provided the inverse relaxation times and the residues associated with a point-source loading for \(l = 2\) and 6. These new modes introduced by the Burgers' body rheology are labelled as \([B_i]\) and there are six of them in this model, with two layers having Burgers' body rheology.

The strength carried by the individual mode is determined by the ratio of the residue to the inverse relaxation time. Examination of these new modes shows that the modes B3, B4, B6 and B7 (see Table 2) carry more strength than the other old Maxwell modes MO (mantle), L (lithosphere), C (core) and M1 (mantle discontinuity). These powerful transient modes have inverse relaxation times spanning between \([10^2\text{ yr}]\) to \([10^4\text{ yr}]\) for models in which the steady state, upper mantle viscosity \(\nu_{\text{UM}}\) is \(10^{21}\) Pa s and the steady lower mantle viscosity \(\nu_{\text{LM}}\) is \(2 \times 10^{21}\) Pa s. Yuen et al. [1986] have also found that the old Maxwell modes are strongly perturbed by the introduction of the transient rheology into the lower mantle. It is important to point out that most of these transient modes are global in character, as described by their eigenfunction distributions.

There are now observed long-term changes in glacier volume which yield mass-balance models of the transfer of water from glaciers excluding those in Greenland and Antarctica, to the surrounding oceans [Meier, 1984]. Although these estimates still need improvements, the uncertainty should be no greater than 50%. Such an amount would not seriously impact the inference of mantle viscosity by a similar factor, as we are still in the linear regime in terms of sensitivity analysis.

Data on glacier balance and ice volume changes for the period 1900–1961 [Meier, 1984] can be found for the thirty-one glacier areas. From the individual rate of growth, the mass \(M_t\) disintegrated over the time period \(a\) (see Figure 1), can be calculated. A period \(a = 200\) yr is used for extrapolating the results into the 21st century. We have checked that the total mass melted over this 61-year period results in a sea level rise of 0.46 mm/yr by using a scaling factor between the average mass balance and the average annual amplitude of each glacier [Meier, 1984]. For these timescales we have assumed a closed hydrological cycle of the glacial process.

The 31 glaciers tabulated in Meier [1984] are employed as point-source forcings, in which the geographical coordinates are supplied as part of the forcing function. The surface den-
ogy is used throughout in this paper.

VLM = \begin{align}  
\sum_{j=1}^{M} P_j(\cos \theta_0) \left[ -\frac{(1 + k_2^0)H(t + b)}{a} + \sum_{j=1}^{M} k_j^j \frac{e^{as_j(t-b)} - 1}{s} H(t - b) \right] 
\end{align}

where \( B = 0.23 \) is a scaling factor in the glacial mass balance [Meier, 1984], \( k_2^0 \) is the elastic Love number and to the set of \( k_j^j \) belong the viscoelastic residuals to the loading problem. The number of glaciers is denoted by \( L \) and the number of normal modes by \( M \).

The other rotational signature, which may be excited by glacial forcings, is the polar wander [Dickman, 1977]. It was found by Gasperini et al. [1986] that polar wander is not excited too much by recent glacial retreats for both Maxwell and Burgers’ body rheologies. This, however, is not the case with temporal variations of the harmonic coefficients of the gravity field, which is the focus of the present study. In Figure 2 we study the sensitivity of the \( J_2 \) excitation to variations of the time span \( b \) in the forcing function of Figure 1. We include all of the glaciers in this set of calculations. The solutions are shown from \( t = 0 \) (1900 A.D.) to \( t = 200 \) yr (2100 A.D.). No significant deviations from the \( b = 500 \) yr solution are encountered until \( b \) is reduced to less than one hundred years. Two different rheologies are considered here. Dashed curves represent Maxwell models with the upper mantle viscosity \( \nu_{UM} = 10^{21} \) Pa s and the lower mantle viscosity \( \nu_{LM} = 2 \times 10^{21} \) Pa s. Solid curves portray the predictions of the Burgers’ body model with the upper and lower mantles having transient parameters of \( \nu_2/\nu_1 = 0.5 \) and \( \nu_2/\nu_1 = 0.1 \). The long-term viscosities are the same as in the Maxwell cases. It is important to note that the effects of the rise in sea-level from glacial melting are to cause a decrease in the spin-rate with an attendant increase in \( J_2 \) which is opposite in sign to the observed value and has about one-third of the observed magnitude [Yoder et al., 1983; Gasperini et al., 1986] It is noteworthy to point out that transient effects are quite obvious for

\begin{align}  
I_{33}^x(s) &= -\frac{3}{2} \int_0^{\pi} \int_0^1 P_2(x, \phi) \phi \ d\phi \ dx 
\end{align}

where for \( f(t) \) shown in Figure 1

\begin{align}  
f(s) &= \frac{1}{s} \exp(-bs) \frac{1}{as^2} 
\end{align}

The viscoelastic loading Love number associated with the degree two harmonic is denoted by \( k_2^x(s) \), which consists of the elastic Love number and all of the transient creep contributions from thirteen viscoelastic modes in this model (see Table 2). The apparent temporal variation in the \( J_2 \) component of the Earth’s gravity field [Yoder et al., 1983] has been shown to be a powerful test of mantle viscosity models [Pel­lier, 1983; Yuen and Sabadini, 1985]. From potential theory one can relate changes in \( J_2 \) to the \( I_{33}^x \) component of the perturbed inertia tensor. It is given in the \( s \) domain by

\begin{align}  
\delta J_2(s) &= \frac{3}{2} I_{33}^x(s) M \frac{1}{a^2} 
\end{align}

where \( M \) is the mass of the Earth. The mathematical description in the time domain for the trajectory of \( J_2 \) can be derived from the formalism previously given in Sabadini et al. [1984]. For the forcing given in Figure 1, this takes the form:

\begin{align}  
J_2(t) &= B \left[ \sum_{j=1}^{L} P_j(\cos \theta_0) \left[ -\frac{(1 + k_2^0)H(t + b)}{a} + \sum_{j=1}^{M} k_j^j \frac{e^{as_j(t-b)} - 1}{s} H(t - b) \right] \right] 
\end{align}

Fig. 1. Schematic diagram of normalized glacial forcing function \( f(t) \). Normalization is with respect to the total mass accumulated by each glacier after a period \( a = 200 \) yr. The times \( t_0 \) and \( t_0 \) are set respectively 1400 and 1900 A.D. with \( b = 500 \) years in the bulk of the calculations presented here.

One prominent response of the planet to ice sheet melting is the change in the moment of inertia. In the coordinate system where the \( x_3 \) axis pierces the surface location of the glacier, the perturbation of the inertia tensor element \( I_{33}^x \) in the transformed domain is given by [Sabadini et al., 1982; Yuen et al., 1982] as

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this time window, as the differences between Maxwell and Burgers’ body models become larger with time, especially for larger values of $b$.

In Figure 3 we illustrate how the $J_2$ data can be satisfied by using Pleistocene deglaciation as the forcing for both Maxwell and Burgers’ body rheologies. The difference in the behavior of the various Burgers’ body rheologies is also displayed for this situation in which recent glacial events are not included. The glacial forcing due to the last ice age’s termination is modelled by an amount of global sea level rise which is equivalent to 90 m from the two northern hemisphere ice sheets and 25 m from Antarctica’s retreat [Yuen et al., 1982]. The upper mantle in all these models has a Maxwell rheology with $v_1 = 10^{21}$ Pa s. For long wavelength deformation we find small differences in the time history of $J_2$ signature between a two-layer Burgers’ body model and a two-layer Maxwell-Burgers’ model, since the eigenfunctions for the $l = 2$ harmonic sample the lower mantle much more thoroughly than the upper mantle. One observes that with the experimentally constrained values of $\mu_2/\mu_1 = 0.1$ and $v_2/v_1 = 0.5$ [Sabadini et al., 1987; Smith and Carpenter, 1987] the solution with a lower mantle viscosity of $10^{22}$ Pa s (dotted curve) comes closer than a solution (curve 1) with a steady, lower-mantle viscosity of $2 \times 10^{21}$ Pa s. A better fit can be obtained by going back to the previously used [Yuen et al., 1986] parameters of $\mu_2/\mu_1 = 0.1$ and $v_2/v_1 = 0.1$ (see curve 2). The existence of another solution [Yuen et al., 1982] with a higher, lower-mantle viscosity ($3 \times 10^{23}$ Pa s) is shown by the dashed dotted curve with a nearly flat trajectory. A Maxwell model (dashed curve) with $v_{1M} = 2 \times 10^{21}$ Pa s has been plotted for comparison and is observed to be quite close to the Burgers’ body model with a higher steady state lower mantle viscosity ($v_{1M} = 3 \times 10^{23}$ Pa s). It is obvious that in order to discriminate better among the different viscosity solutions, it is essential either to reevaluate the ancient lunar eclipse data at an earlier historical time [Stevenson and Morrison, 1984], or to wait another 1000 years in the future. More points are needed for constraining the multiple viscosity solutions. New datum concerning the second derivative of the $J_2$ temporal variation, i.e., $J_4$, may also prove to be a very strong discriminant between low and high viscosity solutions in the lower mantle.

Fig. 4. The excitation of $J_2$ by glacial meltings. The upper mantle has a Maxwell rheology with $v_{1M} = 10^{21}$ Pas. The lower mantle has a Burgers’ body rheology with $v_{2M} = 4 \times 10^{21}$ Pas. We vary $v_2/v_1$ in panel 4a with $\mu_2/\mu_1$ kept constant at 0.1. In panel 4b $v_2/v_1$ is maintained at 1.0 while $\mu_2/\mu_1$ is varied from 0.05 to 0.5 without any effects.

3. $J_2$ EXCITATION FROM RECENT GLACIAL FORCING

Within the context of a steady (upper mantle) and a transient (lower mantle) model, we illustrate in Figure 4 the effects of varying $v_2/v_1$ and $\mu_2/\mu_1$ in the Burgers’ body rheology on the $J_2$ signal excited by recent glacial melting. These parameters are varied because there are still some uncertainties in the transient rheological parameters $\mu_2$ and $v_2$. It is found here that variations of $v_2/v_1$ are much more influential than those involving $\mu_2/\mu_1$. Solutions with $v_2/v_1$ close to unity behaves like a Maxwell model. Solutions with smaller $v_2/v_1$ display larger slopes in $J_2$. As a first illustration of the application of the two-layer Burgers’ body model, whose eigenspectral characteristics are displayed in Table 2, we show in Figure 5 a sequence of predicted $J_2$ curves from glacial excitation. These comparisons again point out the greater importance of the $v_2/v_1$ ratio to that of $\mu_2/\mu_1$. Smaller $v_2/v_1$ induces higher rates of decay, with discernible differences appearing after 150 yr. Even with such a small value of $v_2/v_1$ the smallest $J_2$ signal is still about 20% of the observed value some 200 years after the onset of the current melting episode (cf. Figure 5a). For $v_2/v_1$ close to one [Smith and Carpenter, 1987] even lowering $\mu_2/\mu_1$ to 0.05 does not grossly change the character of the $J_2$ perturbation (cf. Figure 5b). The effects of varying only the transient parameters of the lower mantle are shown in Figure 6. From comparing Figures 5a and 6 it is clear that very little difference is

Fig. 5. Glacial excitation of $J_2$. The entire mantle has Burgers’ body rheology with long-term viscosities of $v_{1M} = 10^{21}$ Pas and $v_{2M} = 4 \times 10^{21}$ Pas. In panel 5a $v_2/v_1$ is varied, while in panel 5b $\mu_2/\mu_1$ is changed.

Fig. 6. $J_2$ time history from glacial forcings. The long-term viscosities in lower and upper mantles are the same as in Figure 5. The Burgers parameters of $v_2/v_1 = 0.5$ and $\mu_2/\mu_1 = 0.1$ are used in the upper mantle. In the lower mantle $v_2/v_1$ is varied with $\mu_2/\mu_1 = 0.1$ used throughout.
produced by selectively varying the rheological parameters of the lower mantle. The importance of \( v_2/v_1 \) is again emphasized by this figure.

We have examined the possibility of whether the \( J_2 \) excitation by glaciers is also sensitive to the long-term viscosity, particularly in the lower mantle. Figure 7 shows \( J_2 \) predictions for a suite of viscosity models in which the viscosity is varied between \( 10^{20} \) and \( 10^{23} \) Pa s. The time of observation is 80 years following the onset of glacial retreats. Panel (7a) describes a three-layer model with just one viscoelastic mantle layer. Panels (7b) and (7c) portray Maxwell (upper mantle)-Burgers' (lower mantle) and Burgers' (upper mantle)-Burgers' (lower mantle) 4-layer models, respectively. We vary the long-term viscosity of the lower mantle \( v_{LM} \) in these cases. In panel (7d) we change the Burgers' body parameter \( \mu_2/\mu_1 \) in the lower mantle for this Burgers'-Burgers' model. What is to be emphasized are the lower rates of \( J_2 \) found for mantle viscosities (lower mantle viscosities for panels (7b) through (7d)) less than about \( 10^{21} \) Pa s. The rates can increase by a factor of 2 for a stiffer lower mantle with viscosities surpassing \( 10^{21} \) Pa s. Depending on the values of \( \mu_2/\mu_1 \) this rise in \( J_2 \) is delayed until higher values of \( v_{LM} \) are reached (see panel (7d)). This transition takes place at lower values of \( v_{LM} \) for Maxwell rheology. Again, if \( v_2/v_1 \) is lower than experimental values of \( 0(1) \) [Smith and Carpenter, 1987], \( J_2 \) can undergo changes for \( v_{LM} \) between \( 10^{21} \) and \( 10^{22} \) Pa s. These results would argue for more precise determinations of transient rheological parameters for better constraining deep mantle viscosity.

We show in Figure 8 a temporal sequence of \( J_2 \) signatures as a function of the longterm, lower mantle viscosity. The main point of these predictions is that it is feasible to discriminate different lower mantle viscosity solutions by a continual monitoring of the secular rotational responses due to recent glacial discharges. It is also crucial for experimentalists to place more stringent bounds on \( v_2/v_1 \) of candidate lower mantle substances.

Studies of current mass balance of the large Antarctic ice sheet [DOE, 1985] suggest that it may be growing and taking water from the world oceans. As the Antarctic ice sheet lies close to the rotational axis, any significant mass exchange between the continent and ocean would have a strong effect on \( J_2 \). Calculations of \( J_2 \) excitation due to growth of the Antarctic ice sheet have been carried out in which the mass accumulation is modelled as a point-source, located at the south pole. We have also modelled it as a finite disk with angular radius \( z = 20^\circ \) and found that, for \( l = 2 \) responses, a point-source is quite adequate for practical purposes. In Figure 9 are shown the \( J_2 \) signatures produced by Antarctic ice sheet growing at a rate of \(-0.3\) mm/yr in the global sea level [DOE, 1985]. The time history of this growth is that this phase began in 1900 A.D. and since that time proceeded to grow at a constant rate. Since the axial moment of inertia decreases as a result of mass being accumulated near the pole, the growth of Antarctica excites \( J_2 \) with the same negative sign (see equation (5)) as for the Pleistocene melting [Yoder et
Fig. 9. $J_2$ excitation as a function of lower mantle viscosity for Antarctic growth. A rate of $-0.3 \text{ mm/yr}$, in sea level is used. Panel (a) has a Maxwell upper mantle with $v_{LM} = 10^{21} \text{ Pas}$ and $\mu_2/\mu_1 = 0.1$ in lower mantle with $v_2/v_1$ varying. The four-layer Burgers' body model has upper mantle creep parameters the same as in Figure 8; lower mantle with $\mu_2/\mu_1 = 0.1$ is used in the lower mantle with $v_2/v_1$ varying.

Fig. 10. Schematic diagram of the combination of the $J_2$ signature by recent cryospheric activities. Dark strip represents data to be inverted, if there are not other sources of excitation except for Pleistocene deglaciation. The dotted and striped strips represent, respectively, the contribution due to the last ice age because of contaminations due to present-day activities. The symbols L and U denote the lower and upper branch viscosity solutions in a Maxwell earth in which only the last deglaciation is used as the forcing. $L_A$, $L_D$, $U_D$, and $U_A$ are the perturbed viscosity solution in view of these new forcings. For $J_2$, $U_A$ and $U_D$ and $L_A$ and $L_D$ have opposite tendencies. The symbols A and G represent Antarctic and glacial contributions. For Burgers' body rheologies the nature of the multiple solutions is far more complicated [Yuen et al., 1986].

Fig. 11. Temporal evolution of the degree 2 tidal Love number in the first century after a static forcing is applied. In all cases the upper and lower mantle, steady state viscosities are respectively $10^{24}$ and $2 \times 10^{21} \text{ Pas}$. The upper mantle's Burgers body parameters are $v_2/v_1 = 0.75$ and $\mu_2/\mu_1 = 0.1$. Only the lower mantle's viscosities transient parameters have been varied. Curve M denotes a Maxwell four-layer model.
less than 20% for the first 100 years of excitation for \( v_{LM} = 10^{21} \) Pa s and \( v_{LM} = 2 \times 10^{21} \) Pa s. For the Burger's body models in which the same \( v_2/v_1 \) and \( \mu_2/\mu_1 \) are used in the upper and lower mantles, one finds that lower values of \( v_2/v_1 \) produce the largest amounts of viscoelastic dispersion. This is about 25% after 20 years and reaches 78% after a century. Dispersion, attained for \( v_2/v_1 = 0.1 \) and \( \mu_2/\mu_1 = 0.1 \), is about 62% of its ultimate fluid tidal number value \( k_2^p(t = \infty) \). These curves in Figure 11 show definitely that there are significant differences in the prediction between Maxwell and Burger's body rheologies, based on laboratory extracted parameter values [Smith and Carpenter, 1987], four timescales less than 100 years. Future satellite measures of long period tidal Love numbers for the low degree harmonics will be of extreme value in constraining the parameter values of short-term mantle rheology.

4. EXCITATION OF HIGHER ZONAL HARMONICS \( J_j \)

In this section we will give an explicit derivation of the zonal harmonic contribution, starting from a general spherical-harmonic expansion. It has been shown by Alexander [1983] that, for certain satellite orbit effects, higher degree harmonic terms can contribute to the perturbations of satellite trajectories by Pleistocene glacial rebound. We will now examine the perturbations to the higher zonal harmonics \( J_j \) from recent glacial activities and also from the potential growth of the Antarctic ice sheet. We will use the Green's function approach in deriving the equations.

From solving the coupled viscous-gravitational equations [Sabatini et al., 1982], one can construct the non-dimensional Green's function for the geoid perturbation for a point-source, impulsive load placed on the surface. It is given by

\[
J(\gamma) = \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} J_{jl}(\gamma) \text{P}_l(\cos \gamma) \tag{7}
\]

where \( \gamma \) represents the angular distance from the pole (source location) and is a function of both the colatitude and longitude. We define the time-dependent spectral coefficient of the geoid to be

\[
J_{j}(t) = \frac{1}{M} \left( 1 + k^0_j + \sum_{l=0}^{\infty} k_j^l e^{at} \right) \tag{8}
\]

From this Green's function (equation (7)) one may obtain the change in the geopotential due to surface loading from all of the active glaciers as

\[
\delta J(d, \theta, \phi, t) = \sum_{j=1}^{N} \int \int d\Omega \sigma_j(\theta - \theta', \phi - \phi') f(t') J(t) - \phi(t') \tag{9}
\]

Next we expand the point-source term of each glacier (equation (2)) and the Green's function (equation (7)) in spherical harmonics and then integrate over the angular part to obtain

\[
\delta J(d, \theta, \phi, t) = \sum_{j=1}^{N} M_j \int_{0}^{\infty} dt' f(t') J_j(t) - \phi(t') \tag{9}
\]

where we define \( J_j \) is equal to \( \delta J/dt \). The elastic loading Love number of degree \( l \) is denoted by \( k_i^l \) and the set of \( \{k_i^l\} \) represents the residuals which contribute to the relaxation of degree \( l \) deformation. The colatitudes of the glaciers are given by \( \theta_i \).

In Figure 12 we show the temporal evolution of the higher degree perturbations up to \( l = 9 \) and compare them with the degree two contribution. These calculations are done for current glacial forcings. One observes that the \( J_j \) component dominates over the other harmonics, as in the case of Pleistocene deglaciation [Yoder et al., 1983; Alexander, 1983]. Both \( J_3 \) and \( J_5 \) have the same sign as for the forcing from the last ice age. It is interesting to note that the magnitudes of \( J_3 \) and \( J_5 \) predicted by these models fall within the precision of the proposed GRM mission [Wagner and McAdoo, 1986]. The relative rates of decay for the different rheologies depend on the degree of the harmonic. The amount of excitation of the higher harmonics by present-day glacial forcing is a significant fraction of the predicted \( J_3 \) due to the last deglaciation [Yoder et al., 1983]. For glacial retreats, zonal harmonics of degree \( l = 4, 5, 7 \) and 8 are small and not included in the figure. For higher harmonics the differences between Burgers' body model (solid curves) and Maxwell rheology are slight, in comparison to those \( J_3 \) and \( J_5 \).

Contemporary melting of the polar ice caps and glaciers around the earth will result potentially in detectable variations of the harmonic coefficients of the earth's geopotential but not in the drift of the pole [Gasperini et al., 1986]. The sensitivity of the higher harmonics to variations of the long-term, lower
mantle viscosity has not been demonstrated previously within the context of the Burgers' body model. We show in Figures 13 and 14 the excitation of the higher harmonics by glacial retreats (Figure 13) and by the possible growth in the Antarctic ice cap (Figure 14). For higher harmonics we have employed a finite spherical ice-cap with an angular amplitude of 20% for modelling Antarctica. A closed hydrological cycle is obtained by assuming that there is a deficit of mass in a global ocean outside Antarctica with $-M_a$, where $M_a$ is the total mass accumulated up to time $t$ from $t = 0$ (1900 A.D.) in Antarctica. For a time function which assumes a constant growth rate, we can derive the viscoelastic responses of the gravity coefficients due to recent Antarctic ice cap growth by the same procedures used previously in obtaining (12). It is given by

$$J_l(t) = \frac{(-1)^l M_a}{r(l + 1)M} \frac{\partial}{\partial t} \frac{P_l(\cos \alpha)}{\cos \alpha} \left[ \frac{(1 + k_j^2)H(t)}{t} + \sum_{j=1}^{M} \frac{k_j^l}{5j^l} (\epsilon^{2j} - 1)H(t) \right] \quad (13)$$

Inspection of Figures 13 and 14 shows that Antarctic ice volume variability is much more efficient than glacial retreats in its ability to distinguish lower-mantle viscosity from the excitation of higher harmonics, since the perturbations are more discernable. The magnitudes of $J_3$, $J_4$, and $J_6$ are only slightly smaller to those examined for Pleistocene deglaciation [Yoder et al., 1983]. For recent glacial events only $J_4$ exhibits significant variation with $v_{LM}$. On the other hand, in the case of Antarctic forcing even as high a degree as $l = 9$ is able to feel variations of the lower mantle viscosity. It is of importance to note that in contrast to $l = 2$, some of the higher harmonics ($l = 3$ and $6$) from the two forcings reinforce each other. Such an enhancement would make it more difficult to separate out the contributions due to Pleistocene deglaciation from the current forcings, when secular variations of higher harmonics are analyzed in the near future. The same sensitivity of the higher solutions to the viscosity ratio $\nu_2/\nu_1$ is found. However, we must be cognizant of the fact that there are still enormous uncertainties in the polar ice cap estimates. Still the possibility is now open for additional independent estimates of lower mantle viscosity by the use of higher gravity harmonics from satellite observations. Higher harmonics can be measured more accurately by lowering the orbit as in the case of the Starlett satellite [Alexander, 1983].

5. DISCUSSIONS AND CONCLUSIONS

On the basis of the results of these calculations, we argue for the need to monitor current cryospheric activities in conjunction with modern methods of space observation to address questions in mantle rheology that previously have relied on signatures produced by the last ice age or by geoid anomalies. It is of extreme importance to separate out the relative contributions to the temporal variations of the gravity field made by the current cryospheric forcings, and that due to the last deglaciation. The precise determination of perturbations to higher harmonics of the gravity field would permit independent estimates of mantle viscosity, since the various harmonics are excited differently. These calculations show the need for acquiring more transient creep data to be used in the Burgers' body rheology. It is found possible to distinguish certain distinct features of long-term viscosity in the lower mantle from monitoring satellite motions for a timespan between 10 and 100 years, provided we know better the parameter $\mu_2/\mu_1$ and $\nu_2/\nu_1$, associated with transient and steady-state creep in the Burgers' body rheology. It is also encouraging to see that the required precision of the spherical harmonic coefficients, $O(10^{-12})$ after six months of observations, is attainable with GRM tracking capabilities in the not too distant future [Wagner and McAdoo, 1986].

By means of transient viscoelastic modeling we have demonstrated that the longwavelength components of the Earth's
gravitational field are sensitive to current glacial discharges [Meier, 1984] and also to the growth of the Antarctic ice sheet occurring today [DOE, 1985]. Corrections to the $J_2$ value currently attributed solely to the Pleistocene deglaciation may be as large as 30%, depending on the magnitude of growth of the Antarctic ice sheet. These effects would cause some uncertainties, no more than a factor of two or three, in the lower mantle viscosities estimated from the $J_2$ data [Peltier, 1983; Yuen and Sabadini, 1985] for the longer branch solutions. Although there are uncertainties in current glacial melting estimates, the contamination of the inverted viscosity would be linearly proportional to the uncertainties in the input parameters, for errors in the small amplitude regime, 0(10%). Larger uncertainties would require detailed sensitivity analysis for assessing the impact on the inferred viscosity solutions. Our results for the higher zonal harmonics reveal that Antarctica’s mass balance may conceivably play an important role. Since the ice dynamics of Antarctica are linked strongly to the important question of global sea level rise, every effort should be made to collect all of the observational evidence by satellite altimetry, satellite imaging and space geodesy, which might be brought to bear on it.

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REFERENCES


Wagner, C. A., and D. C. McAdoo, Time variations in the Earth’s


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